Derivatives & Shapes of Graphs:

- f is increasing on $I \Leftrightarrow f'(x) > 0$, for any $x \in I$

  $\Leftrightarrow$ slope of tangent is +ve.

- f is decreasing on $I \Leftrightarrow f'(x) < 0$, for any $x \in I$

  $\Leftrightarrow$ slope of tangent is −ve.

Ex. Let $f(x) = 2x^3 - 3x^2$

(a) Find the critical pts.

(b) Find the intervals, where f is increasing/decreasing.

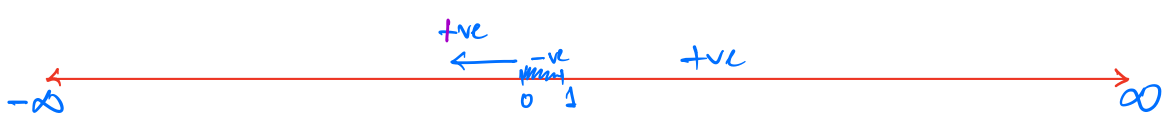
(a) $f'(x) = 0$ (or DNE) implies

$[2x^3 - 3x^2]' = 0$ ie, $6x^2 - 6x = 0$

$\Rightarrow 6x(x-1) = 0$

$\Rightarrow x = 0$ or $x = 1$.

(b)



So, f is increasing on $(-\infty, 0) \cup (1, \infty)$

f is decreasing on $(0, 1)$

Can we draw the graph of f with the above information?

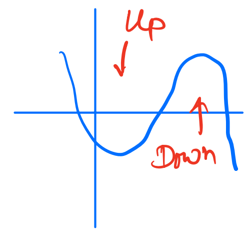
NO

Concavity (Up & Down)

f be a differentiable function on I. We say

(a) Concave Up : f' is increasing on I

(b) Concave Down : f' is decreasing on I.



Working Method :

① 'f' Concave Up on I, when $f''(x) > 0$, for all $x \in I$.

② 'f' Concave Down on I, when $f''(x) < 0$, for all $x \in I$.

Note :- When f (continuous) changes Concavity, we call that point inflection point of f. $[f''(x) = 0]$

Eg. $f(x) = 2x^3 - 3x^2$

$f'(x) = 6x^2 - 6x$

$f''(x) = 12x - 6$

So, our pt of inflection is given by $f''(x) = 0$

$\Rightarrow 12x - 6 = 0$

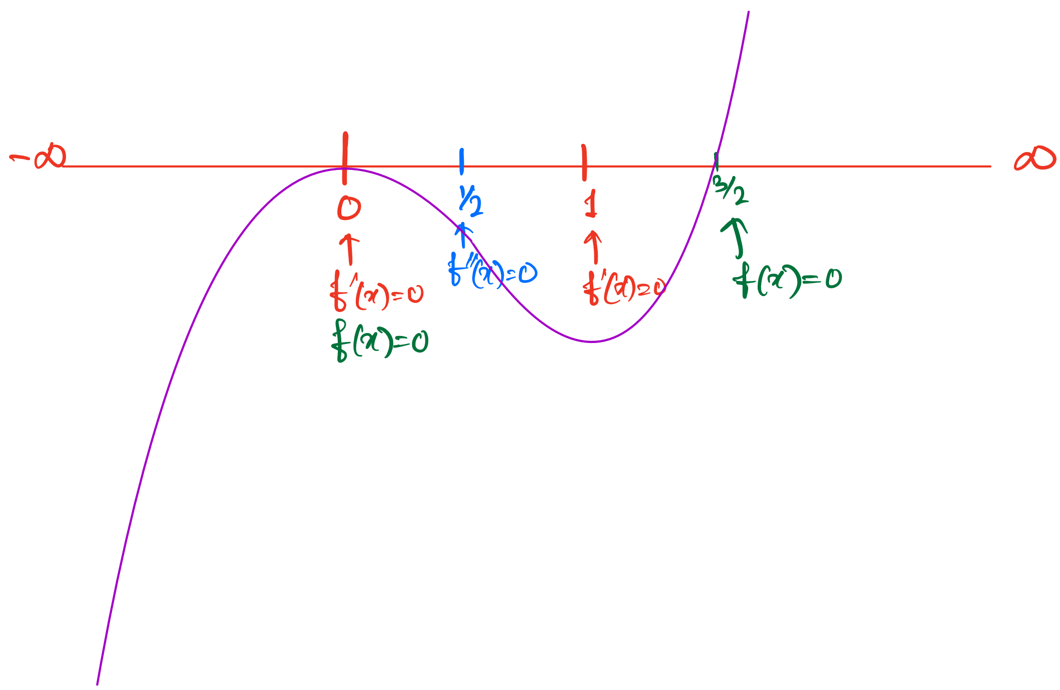
$\Rightarrow x = \frac{1}{2}$.

On left of $x = 1/2$, ie, on $(-\infty, \frac{1}{2})$, $f''(x) < 0$

Concave Down

On right of $x = 1/2$, ie, on $(1/2, \infty)$, $f''(x) > 0$

Concave Up.



$-\infty$
∞
0
1/2
1
3/2
$f'(x)=0$
$f(x)=0$
$f''(x)=0$
$f'(x)=0$
$f(x)=0$